Analytical Expression for the Bias and Frequency-Dependent Capacitance of MOS Varactors

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***Abstract*—Relaxation time approximation is used to obtain an analytical expression for the frequency dependence of the capacitance of MOS varactors that are associated with the inertia of inversion layer formation.**

***Index Terms*—Frequency dependence, MOS varactor, relaxation time approximation (RTA).**

THE DYNAMICS of inversion layer formation is among the factors that determine the frequency response of MOS varactors and has been the subject of two recent investigations [1], [2]. In particular, it was shown that relaxation time approximation (RTA) is sufficiently accurate for most engineering applications and can be readily implemented in SPICE-type circuit simulators. Essentially, RTA is a simple engineering approximation to the detailed microscopic theory that was developed previously (see [3] and [4], and the references therein). In this note, we show that, for the important case of small-signal analysis, one can obtain an exact analytical solution of the relaxation time equation that is in perfect agreement with the results of numerical simulations in [1] and [2].

Let voltage V_g that is applied to the gate of a MOS varactor be given by

$$V_g = V_Q + \mathrm{Re}(\Delta V \cdot e^{j\omega t}) \tag{1}$$

where V_Q is the dc bias, and ΔV is the complex amplitude of the small harmonic component with angular frequency ω. Denote the actual value of the inversion charge per unit channel area as q_i and its quasi-static equilibrium value (assuming instantaneous response to the applied voltage) as q_{i0}. Then, in the RTA [1], [2]

$$\frac{dq_i}{dt} = -\frac{q_i - q_{i0}}{\tau} \tag{2}$$

where τ is the relaxation time, and physical signs are used for all charges. It is convenient to set $q_i = q_{\mathrm{iQ}} + \Delta q_i$ and $q_{i0} = q_{\mathrm{iQ}} + \Delta q_{i0}$, where q_{iQ} is the quiescent point value of q_i corresponding to the dc bias with $V = V_Q$. It follows from (2) that (in phasor notations)

$$\Delta q_i = \frac{\Delta q_{i0}}{1 + j\omega\tau} \tag{3}$$

while the quasi-equilibrium nature of q_{i0} implies that

$$\Delta q_{i0} = \frac{\partial q_i}{\partial V_g}\Delta V \tag{4}$$

where, here and below, all derivatives are evaluated at the quiescent point.

Apart from the relaxation time (2), the essential physics of the method that was developed in [1] and [2] is that the inertia of the inversion layer formation is transferred to the response of surface potential ψ_s through the equation

$$V_g - V_{\mathrm{fb}} + q_i - \psi_s = s\gamma\sqrt{\psi_s - \phi_t + \phi_t \exp(-\psi_s/\phi_t)} \tag{5}$$

where V_{fb} denotes the flatband voltage, $s = \mathrm{sgn}(\psi_s)$, γ is the body factor, and $\phi_t = k_B T/q$ is the thermal potential. Note that the inertia of the ψ_s response to V_g enters (5) through the q_i term.

The bulk charge corresponding to the surface potential determined from (5) is [1], [2]

$$q_b = -s\gamma\sqrt{\psi_s - \phi_t + \phi_t \exp(-\psi_s/\phi_t)}. \tag{6}$$

From (5) and (6), one concludes that $q_b = q_b(\psi_s)$, while $\psi_s = \psi_s(V_g, q_i)$. Hence

$$\Delta q_b = -C_{\mathrm{hf}}\Delta V + B\Delta q_i \tag{7}$$

where

$$C_{\mathrm{hf}} = -\frac{dq_b}{d\psi_s}\frac{\partial \psi_s}{\partial V_g} \tag{8}$$

and

$$B = \frac{dq_b}{d\psi_s}\frac{\partial \psi_s}{\partial q_i}. \tag{9}$$

Combining (4) and (7), one finds that the gate charge phasor

$$\Delta q_g = -\Delta q_i - \Delta q_b \tag{10}$$

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is given by $\Delta q_g = \tilde{c}\Delta V$, where

$$\tilde{c} = C_{\rm hf} - \frac{\partial q_i}{\partial V_g} \cdot \frac{1+B}{1+j\omega\tau}. \tag{11}$$

Hence, capacitance $C = \text{Re}\,\tilde{c}$ is

$$C = C_{\rm hf} - \frac{\partial q_i}{\partial V_g} \cdot \frac{1+B}{1+(\omega\tau)^2}. \tag{12}$$

In particular, the quasi-equilibrium capacitance $C_{\rm qe} = \lim_{\omega\to 0} C$ is

$$C_{\rm qe} = C_{\rm hf} - \frac{\partial q_i}{\partial V_g} \cdot (1+B) \tag{13}$$

so that, from (12), one finds

$$C = C_{\rm hf} + \frac{C_{\rm qe} - C_{\rm hf}}{1+(\omega\tau)^2} \tag{14}$$

where since q_i and q_b are both normalized to $C_{\rm ox}$, so are C, $C_{\rm hf}$, and $C_{\rm qe}$.

Hence, $C_{\rm hf} = C|_{\omega\tau\gg 1}$ can be interpreted as the "high-frequency capacitance." The expression for $C_{\rm hf}$ follows from (5) and (6) as

$$C_{\rm hf} = \frac{1-e^{-u}}{1-e^{-u}-(2q_{\rm bQ}/\gamma^2)} \tag{15}$$

where $u = \psi_{\rm sQ/}\phi{\rm t}$, and $\psi_{\rm sQ}$ and $q_{\rm bQ}$ are the quiescent point values of the surface potential and bulk charge, respectively.

The expression for $C_{\rm qe}$ is well known [3], [4] but is included here for the sake of completeness and uniformity of notation

$$C_{\rm qe} = \frac{F}{1+F} \tag{16}$$

where

$$F = \frac{s\gamma}{2\sqrt{\phi_t}} \cdot \frac{1-e^{-u}+\Delta(e^u-1)}{[e^{-u}+u-1+\Delta(e^u-u-1)]^{1/2}} \tag{17}$$

and $\Delta = \exp(-2\phi_b/\phi_t)$, where ϕ_b denotes the bulk potential.

Comparison of the analytic solution (14) with the results of the SPICE simulation is shown in Fig. 1, where the substrate doping is $2 \cdot 10^{17}$ cm^{-3} and the oxide thickness is 2 nm. The curve for $\omega\tau = 30$ is very nearly $C_{\rm hf}(V_g)$. As expected, a perfect agreement is achieved. Physically, the frequency dependence of MOS capacitance manifests itself after the development of the inversion layer since the response of the majority carriers is practically instantaneous. This is captured explicitly by (14) since, in accumulation and deep subthreshold operation, $C_{\rm qe} \simeq C_{\rm hf} \simeq C$ do not depend on $\omega\tau$. We note in passing that, while the expressions for $C_{\rm hf}$ and $C_{\rm qe}$ assume uniform doping, no such assumption is made in deriving (14). Similarly, while comparison of (14) with the results of numerical simulation is made for the case of constant τ, the small-signal expression (14) remains valid for the bias-dependent τ. On the other hand, the inertia of the inversion layer formation is only one of the factors that are responsible for the frequency dependence of the varactor capacitance [2]. In conclusion, an exact small-signal solution for the frequency response of MOS varactors has been derived using the RTA and found to be identical to previous numerical simulations.

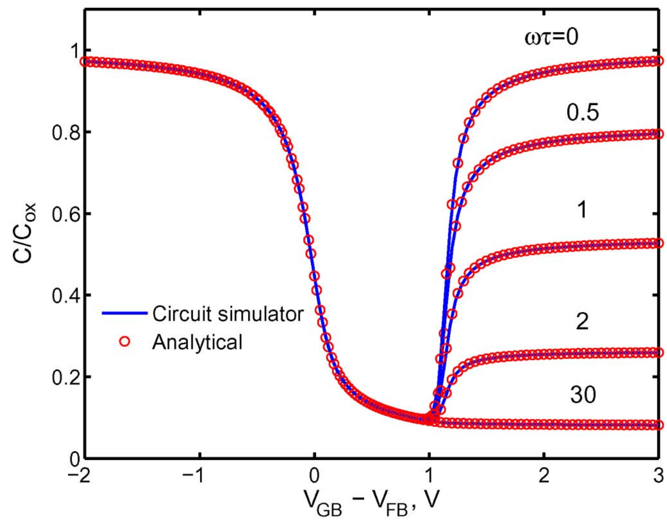

Fig. 1. Comparison of the analytical solution with the circuit simulator results.

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